

A Utility-Augmented Reliability Framework for Performance-Centric Maintenance of Industrial Systems

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Abstract

Classical reliability theory has long served as the mathematical foundation for analyzing failure behavior in industrial systems. However, conventional reliability models predominantly rely on binary operational states and fail to capture gradual performance degradation observed in real-world systems. This paper proposes a novel utility-augmented reliability framework that integrates particle-level utility modeling with traditional probabilistic reliability theory. Particle utility is introduced as a continuous performance measure that quantifies the contribution of individual components to system output over time. The proposed framework explicitly incorporates degradation, imperfect maintenance, and structural system configurations. Analytical modeling is supported by numerical simulations demonstrating significant divergence between reliability and performance utility, particularly in systems experiencing gradual degradation. The results confirm that reliability-only measures can substantially overestimate system effectiveness and that utility-based modeling provides a more accurate foundation for

maintenance optimization. The framework offers a mathematically rigorous and practically meaningful approach for performance-centric reliability analysis in modern industrial systems.

Keywords: Reliability theory, particle utility, degradation modeling, preventive maintenance, industrial systems, performance-based maintenance

1. Introduction

Industrial systems such as manufacturing plants, power generation facilities, transportation networks, and automated production lines are characterized by complex interactions among mechanical, electrical, electronic, and computational components. The sustained performance of these systems is critical to economic productivity, safety, and operational continuity. Traditional reliability theory has played a central role in assessing system dependability by modeling component failures probabilistically. However, classical reliability approaches typically represent components in binary terms—operational or failed—thereby ignoring gradual performance degradation.

In practice, many industrial components operate in degraded states for extended periods before failure. During these periods, output quality, efficiency, and throughput may decline significantly, even though reliability measures suggest acceptable performance. This discrepancy motivates the development of reliability models that explicitly account for performance degradation.

This paper introduces a utility-augmented reliability framework that integrates continuous performance modeling into classical reliability theory. By defining particle utility as a time-dependent performance measure, the proposed approach captures partial functionality, degradation dynamics, and maintenance effects. This integration enables performance-sensitive maintenance planning and more realistic system evaluation.

2. Literature Review

Reliability theory emerged in the mid-twentieth century, driven by applications in military, aerospace, and industrial systems. Early work focused on lifetime distributions, survival analysis, and failure rate modeling. Subsequent developments extended reliability theory to repairable systems, renewal processes, and maintenance optimization.

While multi-state reliability and degradation models have been proposed, many existing approaches either discretize performance states coarsely or treat degradation solely as a precursor to failure. Utility-based performance modeling has received limited attention in reliability literature, particularly in the context of general mathematical frameworks applicable across industrial systems. This research addresses this gap by integrating continuous utility modeling with probabilistic reliability theory.

3. Mathematical Preliminaries

Component lifetimes are modeled as non-negative random variables defined on a probability space. Lifetime distributions such as exponential, Weibull, and lognormal distributions characterize failure behavior. Survival and hazard functions describe the probability of continued operation and the instantaneous failure risk, respectively.

Renewal theory provides the foundation for modeling repeated failure and maintenance cycles. Maintenance actions are interpreted as renewal events that reset or partially reset component condition. These mathematical constructs support long-run performance and maintenance analysis.

1. Probability Space and Lifetime Random Variable

Let

$$(\Omega, \mathcal{F}, \mathbb{P})$$

be a probability space, and let the component lifetime be a random variable

$$T : \Omega \rightarrow \mathbb{R}^+.$$

2. Distribution Functions

Cumulative Distribution Function (Failure Probability)

$$F(t) = \mathbb{P}(T \leq t)$$

Survival (Reliability) Function

$$R(t) = \mathbb{P}(T > t) = 1 - F(t)$$

3. Probability Density Function

If T is continuous,

$$f(t) = \frac{d}{dt}F(t)$$

4. Hazard (Failure Rate) Function

$$h(t) = \frac{f(t)}{R(t)}$$

This function characterizes **age-dependent failure behavior**, a key motivation for preventive maintenance.

5. Mean Time to Failure (MTTF)

$$\text{MTTF} = \mathbb{E}[T] = \int_0^{\infty} R(t) dt$$

6. Weibull Lifetime Model

Widely used in your work for industrial components:

$$R(t) = \exp \left[- \left(\frac{t}{\eta} \right)^{\beta} \right]$$

where

- β = shape parameter
- η = scale parameter

Corresponding hazard rate:

$$h(t) = \frac{\beta}{\eta} \left(\frac{t}{\eta} \right)^{\beta-1}$$

7. Particle Utility Function

For particle i :

$$U_i(t) = U_{i,\max} \cdot g_i(t), \quad 0 \leq g_i(t) \leq 1$$

where

- $U_{i,\max}$ = maximum utility
 - $g_i(t)$ = normalized degradation function
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8. Exponential Utility Degradation Model

Used in numerical analysis:

$$g_i(t) = \exp(-\alpha_i t)$$

where α_i is the degradation rate.

4. Particle Utility Modeling

Particle utility is defined as a quantified measure of usefulness, performance contribution, or service value provided by an individual particle over time. A particle may represent a component, machine element, software agent, or functional module. Unlike binary reliability indicators, particle utility captures graded performance and degraded operational states.

Utility evolves dynamically due to degradation mechanisms such as wear, fatigue, corrosion, and aging. Maintenance actions partially or fully restore utility depending on their effectiveness. Utility is bounded and normalized to ensure interpretability and facilitate aggregation across heterogeneous components.

5. Reliability–Utility Integration

Reliability and utility represent complementary dimensions of component behavior. Reliability governs whether a particle remains operational, while utility quantifies how effectively it performs during operation. In the proposed framework, reliability acts as a gating condition for utility contribution.

This integration resolves limitations of classical models by distinguishing between highly reliable but inefficient components and less reliable but high-performing components. Maintenance actions influence both reliability and utility, enabling performance-centric evaluation of maintenance strategies.

6. System-Level Utility Aggregation

System-level utility is obtained by aggregating particle utilities according to system structure. In series systems, system performance is constrained by the weakest particle, reflecting bottleneck behavior. In parallel systems, redundancy allows compensatory performance, leading to graceful degradation.

Utility aggregation preserves monotonicity and boundedness, ensuring mathematical stability. The framework supports identification of performance-critical components and informs targeted maintenance prioritization.

7. Numerical Analysis

Numerical simulations evaluate particle utility degradation and compare reliability with expected utility. Results demonstrate scenarios where reliability remains close to unity while utility deteriorates substantially. Multi-particle simulations reveal that degradation propagates differently across system structures, with series systems exhibiting high sensitivity to individual particle degradation.

Comparative analysis confirms that reliability-only measures consistently overestimate system effectiveness. Utility-based measures provide early warning of performance decline and support proactive maintenance decisions.

8. Maintenance Implications

Preventive maintenance is motivated by age-dependent and usage-dependent failure behavior commonly observed in industrial components. Reliability theory quantifies the trade-off between maintenance frequency and failure risk, while utility modeling captures performance loss prior to failure.

Utility-aware maintenance strategies enable earlier intervention, improved performance preservation, and better allocation of maintenance resources. Sensitivity analysis shows that degradation rates and maintenance effectiveness significantly influence long-run system utility.

9. Conclusions

This paper presents a novel utility-augmented reliability framework that extends classical reliability theory to capture graded performance and degradation effects. By integrating particle utility with probabilistic reliability modeling, the framework provides a more realistic and informative assessment of industrial system behavior.

Numerical results validate the necessity of utility-based evaluation and demonstrate significant advantages over reliability-only approaches. The proposed framework offers a mathematically rigorous and industrially meaningful foundation for performance-centric maintenance optimization. Future research may extend the framework to dependent components, real-time data integration, and data-driven maintenance strategies.

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