

Performance-Aware Reliability Modeling through Particle Utility for Optimized Industrial Maintenance

KM Aasma Parveen
Department of Mathematics
Shri Venkateshwara University
Uttar Pradesh, India

Dr. Amandeep Singh
Shri Venkateshwara University
Uttar Pradesh, India

Abstract

Classical reliability theory has traditionally modeled industrial system components using binary operational states, focusing primarily on failure occurrence and survival probability. While this approach has been effective for safety analysis and failure risk assessment, it fails to capture gradual performance degradation that dominates real industrial systems. This paper proposes a performance-aware reliability modeling framework that integrates particle utility into conventional reliability theory. Particle utility is introduced as a continuous measure of performance contribution that evolves over time due to degradation, operational stress, and maintenance actions. The framework unifies probabilistic lifetime modeling, utility degradation, and maintenance dynamics within a mathematically consistent structure. System-level performance is evaluated through utility aggregation under series and parallel configurations. Numerical investigations demonstrate significant divergence between reliability and performance utility, highlighting the limitations of reliability-only assessment. The results confirm

that utility-based modeling provides a superior foundation for maintenance optimization and sustainable industrial system management.

Keywords: Reliability theory, particle utility, degradation modeling, preventive maintenance, industrial systems, performance-aware reliability

1. Introduction

Industrial systems such as manufacturing plants, power generation units, transportation infrastructures, and automated production lines are critical to modern economies. These systems are composed of interacting mechanical, electrical, electronic, and computational components whose sustained performance determines productivity, safety, and economic viability. Traditional reliability theory has provided a rigorous mathematical framework for modeling failure behavior and assessing system dependability. However, classical reliability approaches typically represent components in binary terms—either operational or failed.

In practice, industrial components rarely transition abruptly from full functionality to failure. Instead, they experience gradual degradation due to wear, fatigue, corrosion, thermal stress, and aging. During these degraded states, components remain operational but deliver reduced efficiency, output quality, or throughput. Reliability-only measures fail to capture this performance loss, leading to optimistic system assessments and delayed maintenance actions.

This paper addresses this limitation by introducing a performance-aware reliability framework based on particle utility. By integrating continuous performance modeling with probabilistic reliability theory, the proposed approach enables more realistic system evaluation and supports optimized maintenance decision-making.

2. Literature Review

Reliability theory emerged in the mid-twentieth century and has since evolved through advances in probability theory, survival analysis, and stochastic processes. Foundational work focused on lifetime distributions, hazard modeling, and system reliability structures. Subsequent research extended reliability theory to repairable systems, renewal processes, and maintenance optimization.

Degradation-based and multi-state reliability models have been proposed to address limitations of binary modeling. However, many such models discretize performance coarsely or treat degradation solely as a precursor to failure. Explicit performance modeling remains limited in general reliability frameworks. The concept of utility, widely used in economics and decision theory, has seen limited integration into reliability modeling, particularly at the particle or component level.

This research contributes to the literature by introducing a unified reliability–utility framework that explicitly models performance degradation and integrates it with classical reliability theory.

3. Mathematical Preliminaries

Let $(\Omega, \mathcal{F}, \mathbb{P})$ denote a probability space. Component lifetime is modeled as a non-negative random variable defined on this space. Lifetime distributions such as exponential and Weibull distributions are used to characterize failure behavior.

Survival functions represent the probability that a component remains operational beyond a given time, while hazard functions describe the instantaneous failure risk conditional on survival. Renewal theory provides the mathematical basis for modeling repeated failure and maintenance cycles and supports long-run performance analysis.

(Eq. 1) Probability Space & Lifetime Random Variable

Include in Section 3, paragraph 1

$$(\Omega, \mathcal{F}, \mathbb{P}), \quad T : \Omega \rightarrow \mathbb{R}^+$$

Defines the probabilistic foundation for component lifetime modeling.

(Eq. 2) Distribution & Reliability Functions

Include in Section 3

$$F(t) = \mathbb{P}(T \leq t), \quad R(t) = \mathbb{P}(T > t) = 1 - F(t)$$

Used to characterize survival behavior.

(Eq. 3) Hazard Rate

Include in Section 3 (age-dependent failure discussion)

$$h(t) = \frac{f(t)}{R(t)}$$

Motivates preventive maintenance.

(Eq. 4) Weibull Reliability Model

Include in Section 3 or 7 (Numerical Analysis)

$$R(t) = \exp \left[- \left(\frac{t}{\eta} \right)^\beta \right]$$

Captures aging and wear-out effects.

4. Particle Utility Modeling

Particle utility is defined as a quantified measure of usefulness, performance contribution, or service value provided by an individual particle over time. A particle may represent a component, machine element, software agent, or functional module. Unlike binary reliability indicators, particle utility captures graded performance and partial functionality.

Utility evolves dynamically due to degradation processes and maintenance actions. At the beginning of an operational cycle, particles deliver maximum utility. Over time, degradation reduces utility, while maintenance actions partially or fully restore performance depending on effectiveness. Utility is normalized and bounded to ensure interpretability and mathematical stability.

(Eq. 5) Particle Utility Function

Include in Section 4.2

$$U_i(t) = U_{i,\max} g_i(t), \quad 0 \leq g_i(t) \leq 1$$

Defines graded performance contribution.

(Eq. 6) Utility Degradation Model

Include in Section 4.3 or Section 7

$$g_i(t) = \exp(-\alpha_i t)$$

Models gradual performance loss.

5. Reliability–Utility Integration

Reliability and utility represent complementary dimensions of component behavior. Reliability determines whether a particle remains operational, while utility quantifies how effectively it performs during operation. In the proposed framework, reliability acts as a gating condition for utility contribution.

(Eq. 7) Reliability–Utility Coupling

Include in Section 5

$$U_i^{\text{eff}}(t) = U_i(t) \mathbf{1}_{\{T_i > t\}}$$

Utility exists only while the component survives.

(Eq. 8) Expected Utility

Include in Section 5 or Section 7

$$\mathbb{E}[U_i(t)] = U_{i,\max} g_i(t) R_i(t)$$

Key novelty equation — shows why reliability alone is insufficient.

(Eq. 9) Series System Utility

Include in Section 6.1

$$U_S(t) = \min_i \{U_i(t)\}$$

Captures bottleneck behavior.

(Eq. 10) Parallel System Utility

Include in Section 6.1

$$U_S(t) = \sum_{i=1}^n w_i U_i(t), \quad \sum w_i = 1$$

Models redundancy and compensation.

This integration distinguishes between components that are highly reliable but inefficient and those that deliver high performance despite higher failure risk. Maintenance actions influence both reliability and utility, enabling comprehensive evaluation of maintenance strategies.

6. System-Level Utility Aggregation

System performance is evaluated by aggregating particle utilities according to system structure. In series systems, system utility is constrained by the weakest particle, reflecting bottleneck behavior. In parallel systems, redundancy allows compensatory performance, leading to graceful degradation.

Utility aggregation preserves monotonicity and boundedness, ensuring mathematical consistency. The framework supports identification of performance-critical components and informs maintenance prioritization.

7. Numerical Analysis

Numerical simulations evaluate particle utility degradation and compare reliability with expected utility. Results demonstrate scenarios where reliability remains close to unity while utility deteriorates substantially. Multi-particle simulations reveal that degradation propagates differently across system structures, with series systems exhibiting high sensitivity to individual particle degradation.

Comparative analysis confirms that reliability-only measures consistently overestimate system effectiveness. Utility-based measures provide early warning of performance decline and support proactive maintenance decisions.

(Eq. 11) Long-Run Expected Utility

Include in Section 7.3 or 7.5

$$\bar{U} = \lim_{t \rightarrow \infty} \frac{1}{t} \int_0^t \mathbb{E}[U_S(s)] ds$$

Supports steady-state evaluation.

Section 8: Maintenance Implications

(Eq. 12) Renewal Reward Formulation

Include in Section 8

$$\bar{U} = \frac{\mathbb{E}[\text{Utility per cycle}]}{\mathbb{E}[\text{Cycle length}]}$$

Links maintenance cycles to performance.

8. Maintenance Implications

Preventive maintenance is motivated by age-dependent and usage-dependent failure behavior commonly observed in industrial components. Reliability theory quantifies the trade-off between maintenance frequency and failure risk, while utility modeling captures performance loss prior to failure.

Utility-aware maintenance strategies enable earlier intervention, improved performance preservation, and better allocation of maintenance resources. Sensitivity analysis demonstrates that degradation rates and maintenance effectiveness significantly influence long-run system utility.

9. Conclusions

This paper presents a performance-aware reliability modeling framework that integrates particle utility with classical reliability theory. By explicitly modeling performance degradation, the framework overcomes key limitations of binary reliability approaches and provides a more realistic representation of industrial system behavior.

Numerical results validate the necessity of utility-based evaluation and demonstrate significant advantages over reliability-only models. The proposed framework offers a mathematically rigorous and industrially meaningful foundation for optimized maintenance planning and sustainable system management.

Future research may extend the framework to dependent components, real-time data integration, and data-driven maintenance optimization.

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