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Significance Patterns of Eigenvectors in Image Analytics

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Abstract

Eigenvalues play an important role in image processing applications. There are various methods available for image processing. The processing like measurement of image sharpness can be done using the concept of eigenvalues. In case of human face segmentation using elliptical shape, largest and smallest eigenvalue of covariance matrix represent the elliptical shape. Of course you can understand how this will work out only after studying the math behind it. One of the application which is fairly easy to understand is Image Compression (also called dimension reduction). Image compression has been the means of reducing the size of a graphics file for better storage convenience. It is also a useful way of reducing the time requirement of sending large files over the Web via a method in image compression - Principal component analysis (PCA). This technique utilizes the idea that any image can be represented as a superposition of weighted base images.

Keywords: Eigenvector, Eigenvector in Image Processing, Imaging and Eigenvectors

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Introduction

The eigen in eigenvector comes from German, and it means something like "very own." For example, in German, "mein eigenes Auto" means "my very own car." So eigen denotes a special relationship between two things. Something particular, characteristic and definitive. This car, or this vector, is mine and not someone else's. Matrices, in linear algebra, are simply rectangular arrays of numbers, a collection of scalar values between brackets, like a spreadsheet. All square matrices (e.g. 2 x 2 or 3 x 3) have eigenvectors, and they have a very special relationship with them, a bit like Germans have with their cars.

Scenario of Eigenvector in Image Processing

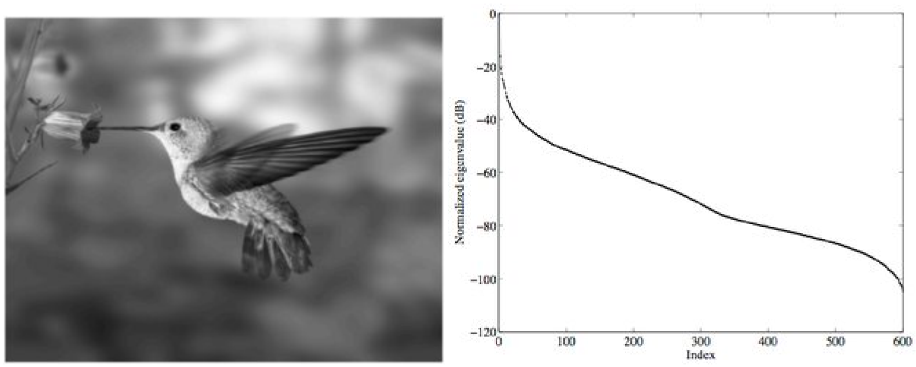

Figure 1: Image Analysis with Eigenvector

Now taking an SVD of the above image gives us 600600 singular values that, when plotted, look like the curve on the right. Note that the yy-axis is in decibels (i.e., 10log10(s.v.)10log10 (s.v.)).

You can clearly see that after about the first 20−2520−25 singular values, it falls off and the bulk of it is so low, that any information it contains is negligible (and most likely noise). So the question is, why store all this information if it is useless?

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Let's look at what information is actually contained in the different singular values. The figure on the left below shows the image recreated from the first 10 singular values (l=10l=10 in J.M.'s answer). We see that the essence of the picture is basically captured in just 10 singular values out of a total of 600. Increasing this to the first 50 singular values shows that the picture is almost exactly reproduced (to the human eye).

Figure 2: Image Analysis Scenario with Eigenvector

Looking at the next 100 singular values (figure on the left), we actually see some fine structure, especially the fine details around the feathers, etc., which are generally indistinguishable to the naked eye. It's probably very hard to see from the figure below, but you certainly can in this larger image. The smallest 300 singular values are complete junk and convey no information. These are most likely due to sensor noise from the camera's CMOS.

Matrices are useful because you can do things with them like add and multiply. If you multiply a vector v by a matrix A, you get another vector b, and you could say that the matrix performed a linear transformation on the input vector.

$Av = b$

It maps one vector v to another, b.

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We'll illustrate with a concrete example. (You can see how this type of matrix multiply, called a dot product, is performed here.)

$$\underset{A}{\begin{bmatrix} 2 & 1 \\ 1.5 & 2 \end{bmatrix}} * \underset{v}{\begin{matrix} 0.75 \\ 0.25 \end{matrix}} = \underset{b}{\begin{matrix} 1.75 \\ 1.625 \end{matrix}}$$

So A turned v into b. In the graph below, we see how the matrix mapped the short, low line v, to the long, high one, b.

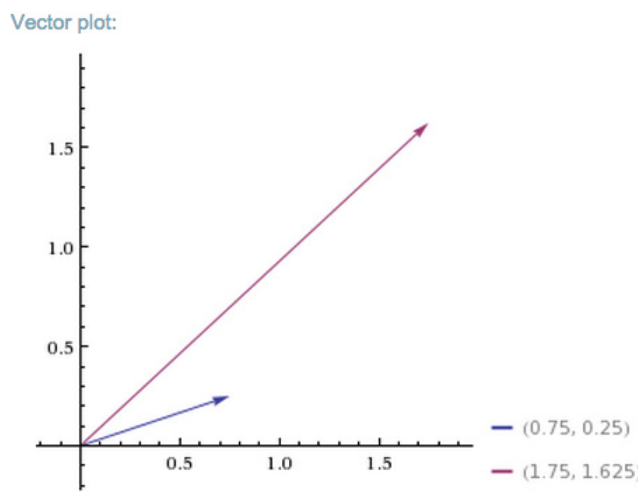

Figure 3: Plot of Values

You could feed one positive vector after another into matrix A, and each would be projected onto a new space that stretches higher and farther to the right.

Imagine that all the input vectors v live in a normal grid, like this:

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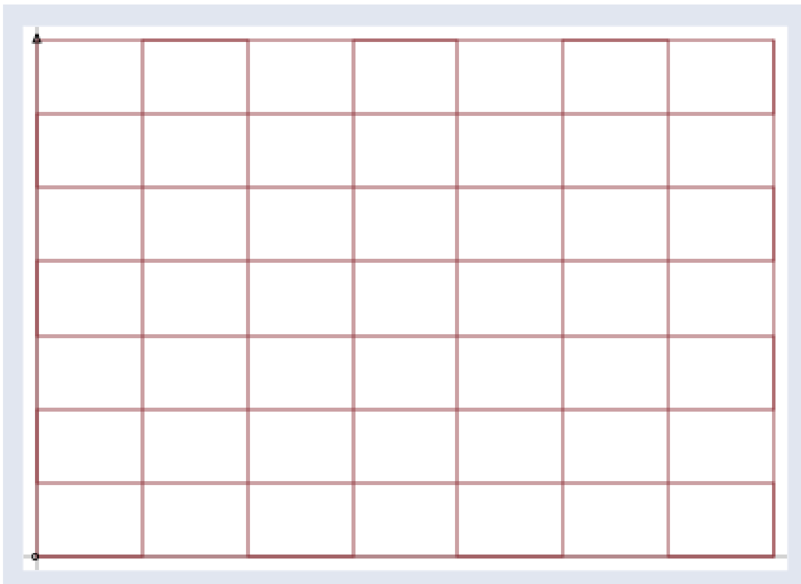

Figure 4: Grid View

And the matrix projects them all into a new space like the one below, which holds the output vectors b:

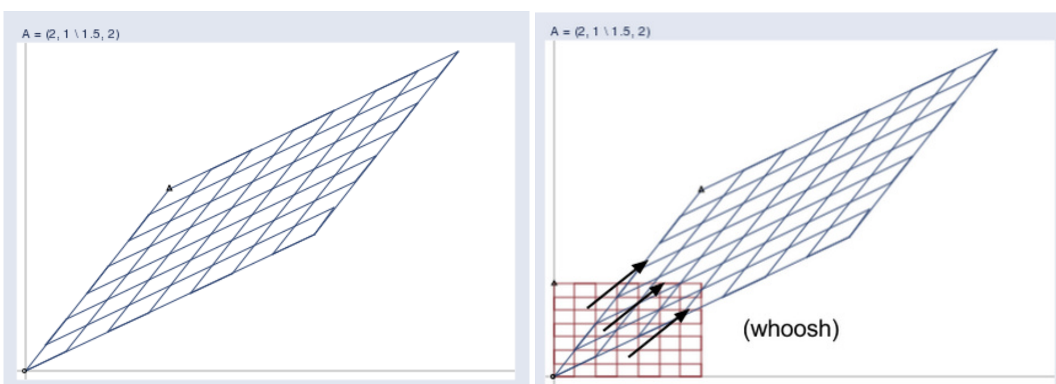

Figure 5: Grid Views

(Source: William Gould, Stata Blog)

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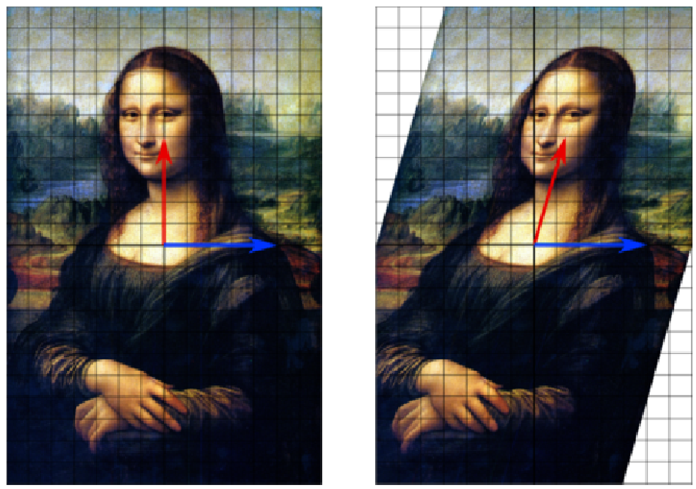
And here's an animation that shows the matrix's work transforming one space to another:
The blue lines are eigenvectors. You can imagine a matrix like a gust of wind, an invisible force that produces a visible result. And a gust of wind must blow in a certain direction. The eigenvector tells you the direction the matrix is blowing in.



So out of all the vectors affected by a matrix blowing through one space, which one is the eigenvector? It's the one that that changes length but not direction; that is, the eigenvector is already pointing in the same direction that the matrix is pushing all vectors toward. An eigenvector is like a weathervane. An eigenvane, as it were. The definition of an eigenvector, therefore, is a vector that responds to a matrix as though that matrix were a scalar coefficient. In this equation, A is the matrix, x the vector, and lambda the scalar coefficient, a number like 5 or 37 or pi.

$$Ax = \lambda x$$

You might also say that eigenvectors are axes along which linear transformation acts, stretching or compressing input vectors. They are the lines of change that represent the action of the larger matrix, the very "line" in linear transformation.

Notice we're using the plural – axes and lines. Just as a German may have a Volkswagen for grocery shopping, a Mercedes for business travel, and a Porsche for joy rides (each serving a distinct purpose), square matrices can have as many eigenvectors as they have dimensions; i.e. a 2 x 2 matrix could have two eigenvectors, a 3 x 3 matrix three, and an n x n matrix could have n eigenvectors, each one representing its line of action in one dimension.1

Conclusion

Because eigenvectors distill the axes of principal force that a matrix moves input along, they are useful in matrix decomposition; i.e. the diagonalization of a matrix along its eigenvectors. Because those eigenvectors are representative of the matrix, they perform the same task as the autoencoders employed by deep neural networks.

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